B, C′, are of course also considered. An equation of Dr. Salmon's is presented in the extended form,

$$\sigma' = 4n(n-2) - 8b - 11c - 2j' - 3\chi' - 2C' - 4B';$$

and it is remarked that σ' denotes the order of the spinode-curve. The Memoir contains an entirely new formula giving the value of β', but some of the constants of the formula remain undetermined.

III. "A Memoir on Cubic Surfaces."

By Professor Cayley, F.R.S. Received November 12, 1868.

(Abstract.)

The present Memoir is based upon, and is in a measure supplementary to that by Professor Schläfli, "On the Distribution of Surfaces of the Third Order into Species, in reference to the presence or absence of Singular Points, and the reality of their Lines," Phil. Trans. vol. cliii. (1863) pp. 193–241. But the object of the Memoir is different. I disregard altogether the ultimate division depending on the reality of the lines, attending only to the division into (twenty-two, or as I prefer to reckon it) twenty-three cases depending on the nature of the singularities. And I attend to the question very much on account of the light to be obtained in reference to the theory of Reciprocal Surfaces. The memoir referred to furnishes in fact a store of materials for this purpose, inasmuch as it gives (partially or completely developed) the equations in plane-coordinates of the several cases of cubic surfaces; or, what is the same thing, the equations in point-coordinates of the several surfaces (orders 12 to 3) reciprocal to these respectively. I found by examination of the several cases, that an extension was required of Dr. Salmon's theory of Reciprocal Surfaces in order to make it applicable to the present subject; and the preceding "Memoir on the Theory of Reciprocal Surfaces" was written in connexion with these investigations on Cubic Surfaces. The latter part of the Memoir is divided into sections headed thus:—"Section I = 12, equation $(X, Y, Z, W)^3 = 0$" &c. referring to the several cases of the cubic surface; but the paragraphs are numbered continuously through the Memoir.

The principal results are included in the following Table of singularities. The heading of each column shows the number and character of the case referred to, viz. C denotes a conic node, B a biplanar node, and U a uniplanar node; these being further distinguished by subscript numbers, showing the reduction thereby caused in the class of the surface: thus $XIII = 12 - B_3 - 2\,C_2$ indicates that the case XIII is a cubic surface, the class whereof is $12 - 7, = 5$, the reduction arising from a biplanar node, B_3, reducing the class by 3, and from 2 conic nodes, C_2, each reducing the class by 2.

	I=12.	II=12−C_2.	III=12−B_3.	IV=12−2C_2. V=12−B_4−C_2.	VI=12−B_3−C_2. VII=12−B_5.	VIII=12−3C_2. X=12−B_4−C_2. XI=12−B_6. XII=12−U_6.	IX=12−2B_3.	XIII=12−B_3−2C_2. XIV=12−B_3−C_2. XV=12−U_7.	XVI=12−4C_2. XVII=12−B_4−2C_2. XIX=12−B_6−C_2.	XVII=12−2B_3−C_2. XX=12−U_8.	XXI=12−3B_3.	XXII=S(1, 1). XXIII=S(1, 1).	
n	3	3	3	3	3	3	3	3	3	3	3	3	n
a	6	6	6	6	6	6	6	6	6	6	6	4	a
δ	0	1	0	2	1	3	0	2	4	1	0	0	δ
κ	6	6	7	6	7	6	8	7	6	8	9	3	κ
b	0	0	0	0	0	0	0	0	0	0	0	1	b
k	0	0	0	0	0	0	0	0	0	0	0	0	k
t	0	0	0	0	0	0	0	0	0	0	0	0	t
q	0	0	0	0	0	0	0	0	0	0	0	0	q
ρ	0	0	0	0	0	0	0	0	0	0	0	1	ρ
j	0	0	0	0	0	0	0	0	0	0	0	2	j
c	0	0	0	0	0	0	0	0	0	0	0	0	c
h	0	0	0	0	0	0	0	0	0	0	0	0	h
r	0	0	0	0	0	0	0	0	0	0	0	0	r
σ	0	0	0	0	0	0	0	0	0	0	0	0	σ
θ	0	0	0	0	0	0	0	0	0	0	0	0	θ
χ	0	0	0	0	0	0	0	0	0	0	0	0	χ
β	0	0	0	0	0	0	0	0	0	0	0	0	β
γ	0	0	0	0	0	0	0	0	0	0	0	0	γ
ι	0	0	0	0	0	0	0	0	0	0	0	0	ι
C	0	1	0	2	1	3	0	2	4	1	0	0	C
B	0	0	1	0	1	0	2	1	0	2	3	0	B
	I.	II.	III.	IV, V.	VI, VII.	VIII, X, XI, XII.	IX.	XIII, XIV, XV.	XVI, XVIII, XIX.	XVII, XX.	XXI.	XXII, XXIII.	
n'	12	10	9	8	7	6	6	5	4	4	3	3	n'
a'	6	6	6	6	6	6	6	6	6	6	6	4	a'
δ'	0	0	0	0	0	0	0	0	0	0	0	0	δ'
κ'	9	9	9	9	9	9	9	9	9	9	9	3	κ'
b'	27	15	9	7	3	3	0	1	3	0	0	1	b'
k'	216	60	18	12	3	0	0	0	0	0	0	0	k'
t'	45	15	6	3	0	1	0	0	1	0	0	0	t'
q'	0	0	0	0	0	0	0	0	0	0	0	0	q'
ρ'	27	15	9	7	3	3	0	1	3	0	0	1	ρ'
j'	0	0	0	1	0	3	0	1	6	0	0	2	j'
c'	24	18	16	12	10	6	8	4	0	2	0	0	c'
h'	180	96	72	38	24	6	12	2	0	0	0	0	h'
r'	30	24	42	17	24	9	32	5	0	2	0	0	r'
σ'	12	12	12	10	9	6	8	4	0	2	0	0	σ'
θ'	0	0	16	0	8	0	16	0	0	0	0	0	θ'
χ'	0	0	0	0	1	0	0	2	0	2	0	0	χ'
β'	54	30	18	13	6	3	0	1	0	0	0	0	β'
γ'	0	0	0	0	0	0	0	0	0	0	0	0	γ'
ι'	0	0	0	0	0	0	0	0	0	0	0	0	ι'
C	0	0	0	0	0	0	0	0	0	0	0	0	C′
B′	0	0	0	0	0	0	1	0	0	1	3	0	B′